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ORIGINAL CONTRIBUTION

Jyh-Ping Hsu
Li-Hsien Yeh
Ming-Hong Ku

Electrophoresis of a spherical particle along the axis of a cylindrical pore filled with a Carreau fluid

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J.-P. Hsu (✉) · L.-H. Yeh · M.-H. Ku
Department of Chemical Engineering,
National Taiwan University,
Taipei 10617, Taiwan
e-mail: jphsu@ntu.edu.tw
Tel.: +886-2-23637448
Fax: +886-2-23623040

Abstract The electrophoresis of a spherical particle along the axis of a cylindrical pore filled with a Carreau fluid is investigated theoretically. In addition to the boundary effect due to the presence of the pore, the influences of the thickness of double layer surrounding a particle and the properties of the fluid on the electrophoretic behavior of the particle are also examined. We show that, in general, the presence of the pore has the effect of retarding the movement of a particle. On the other hand, the shear-thinning nature of the liquid phase is advantageous to its movement. For both Newtonian and Carreau fluids, the mobility of a particle increases monotonically with the decrease in the thickness of double layer, but the mobility is more sensitive to the variation of the thickness of double layer in the latter. The mobility of a particle in a Carreau fluid is larger than that in the corresponding Newtonian fluid, and the difference between the two increases with the decrease in double-layer thickness; in addition, depending upon the values of the parameters assumed, the percentage difference can be in the order of 50%.

Keywords Electrophoresis · Boundary effect · Sphere in cylindrical pore · Non-Newtonian fluid · Carreau model

Introduction

Since the pioneer theoretical work of von Smoluchowski [1] about a century ago, electrophoresis has been studied both theoretically and experimentally in numerous fields. The advances in mathematical tools and computing facilities make the analysis on the phenomenon more general and accurate, extending the original problem of an isolated particle in an infinite fluid medium under conditions such as low surface potentials, thin double layers, and constant surface potentials to more realistic ones. Up to now, relevant studies on electrophoresis remain active in various modern areas.

Previous studies on electrophoresis are mainly focused on the case when the fluid medium is Newtonian, although a non-Newtonian medium is not uncommon in practice in the scope of colloidal science. One of the possible reasons behind this is that solving the governing equations (a set of coupled, nonlinear differential equations) of electrophoresis is nontrivial even for a Newtonian fluid. A non-Newtonian medium can be encountered, for instance, when surfactant or polymer is introduced into a colloidal dispersion to improve its stability [2]. Another typical example is that when the concentration of dispersed particles is high, such as the slurry used in chemical–mechanical polishing, the corresponding dispersion medium becomes non-Newtonian in nature [3]. Lee et al. [4] made the first theoretical attempt on the analysis of electrophoresis conducted in a non-Newtonian fluid by considering a sphere at the center of a spherical cavity filled with a Carreau fluid under the conditions of low surface potentials. The shear-thinning nature of the fluid was found to have the effect of increasing the electrophoretic mobility of a particle. Their model was extended by Hsu et al. [5] to the case where a sphere is at an arbitrary position in a spherical cavity. Hsu et al. [6] investigated the electrophoresis of a concentrated spherical dispersion for the case of

low surface potential and Carreau fluid. Their analysis was extended by Lee et al. [7] to the case of arbitrary surface potential. The electrophoresis of a rigid sphere in a Carreau fluid normal to a planar surface was studied by Lee et al. [8]. A conclusion common to these studies is that the shear-thinning nature of a fluid medium is advantageous to the electrophoresis of a particle.

In this study, the electrophoresis of a spherical particle along the axis of a cylindrical pore filled with a Carreau fluid is analyzed under the conditions of weak applied electric field and low surface potential. In addition to the boundary effect due to the presence of a pore, the influences of the thickness of double layer surrounding a particle and the properties of the fluid medium on the electrophoretic behavior of the particle are also discussed. The problem under consideration has practical applications such as for the removal of charged colloidal contaminants from a porous media [9] and as a potential separation technology [10, 11].

Theory

As shown in Fig. 1, we consider the electrophoresis of a rigid, nonconducting, spherical particle of radius a along the axis of an infinite, nonconducting cylindrical pore of radius b filled with a Carreau fluid. The cylindrical coordinates (r, θ, z) are chosen with its origin located at the center of the particle, and a uniform electric $\mathbf{E}$, which is in the z direction with strength E, is applied. Since the geometry considered is θ symmetric, we consider the (r, z) domain only. Under the conditions of weak applied electric field and low electrical potential, the governing equations for the electric field can be approximated by

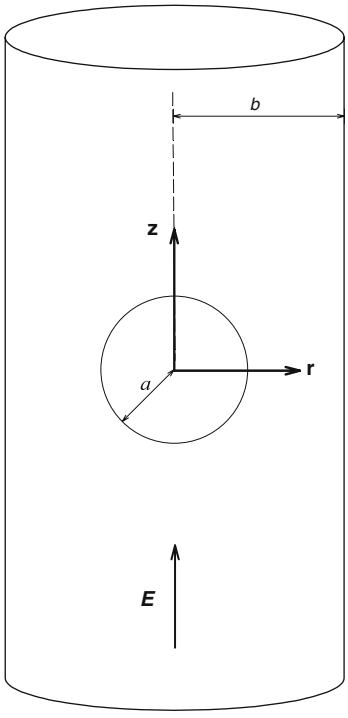

Fig. 1 Schematic representation of the problem considered where a spherical particle of radius a is placed on the axis of an infinite cylindrical pore of radius b filled with a Carreau fluid. A uniform electric field $\mathbf{E}$ parallel to the axis of the pore is applied. The cylindrical coordinates (r, θ, z) with its origin located at the center of the particle are adopted

$$\nabla^2 \Psi_1 = \kappa^2 \Psi_1 \tag{1}$$

$$\nabla^2 \Psi_2 = 0 \tag{2}$$

where $\Psi_1+\Psi_2$ represents the electrical potential of the system under consideration, Ψ_1 is the electrical potential in the absence of the applied electric field, and Ψ_2 is that outside the particle arising from the applied electric field, ∇^2 is the Laplace operator, $\kappa = \left[\sum_j n_j^0 (ez_j)^2 \Big/ \varepsilon k_B T\right]^{1/2}$ is the reciprocal Debye length, n_j^0 and z_j are the bulk number concentration and the valence of ionic species j, respectively, and ε, e, k_B, and T are the permittivity of the liquid phase, the elementary charge, the Boltzmann constant, and the absolute temperature, respectively. The following boundary conditions are assumed for the electric field:

$$\Psi_1 = \zeta_p \text{ and } \mathbf{n} \cdot \nabla \Psi_2 = 0, \text{ on particle surface} \tag{3}$$

$$\Psi_1 = \zeta_w \text{ and } \frac{\partial \Psi_2}{\partial r} = 0, r = b \tag{4}$$

$$\Psi_1 = 0 \text{ and } \nabla \Psi_2 = -\boldsymbol{E}, |z| \to \infty, r < b \tag{5}$$

where $\mathbf{n}$ is the unit outward normal vector. Equations (3) and (4) imply that the electrical potential on the surfaces of the particle and the pore is constant.

Suppose that the liquid phase is incompressible and its viscosity can be described by [12, 13]

$$\frac{\eta(\dot{\gamma})}{\eta_o} = \left[1 + (\alpha \dot{\gamma})^2\right]^{(n-1)/2} \tag{6}$$

where η_0 is the viscosity at zero shear rate, α is the relaxation time constant, and n is the power-law index. Note that if either $n \to 1$ or $\alpha \to 0$, the present fluid becomes a Newtonian fluid, and if α is sufficiently large, it becomes a power-law fluid. Since the Reynolds number is small for

electrophoresis under typical conditions, the flow field at steady state can be described by [4, 12]

$$-\nabla \cdot (\eta \nabla \mathbf{u}) - \nabla p = \rho \nabla (\Psi_1 + \Psi_2) \tag{7}$$

$$\nabla \cdot \boldsymbol{u} = \boldsymbol{0} \tag{8}$$

where $\rho = \sum_j z_j e n_j^0 \exp\left[-z_j e(\Psi_1 + \Psi_2)/k_B T\right]$, and p and $\boldsymbol{u}$ are the pressure and the velocity of the liquid phase, respectively. If we let U be the magnitude of particle velocity in the z direction and $\boldsymbol{e}_z$ be the unit vector in the z direction, then the boundary conditions associated with Eqs. (7) and (8) are

$$\boldsymbol{u} = U\boldsymbol{e}_z \text{ on particle surface} \tag{9}$$

$$\boldsymbol{u} = 0, r = b \tag{10}$$

$$\boldsymbol{u} = 0, |z| \to \infty, r < b \tag{11}$$

Here, we assume that the surface of a particle and that of the cavity are no-slip.

In this problem, only the z components of the forces acting on a particle, including the electrostatic force and the hydrodynamic force, are needed to be considered. The z component of the former, F_E, can be calculated by

$$F_E = \iint_S \sigma E_z dS \tag{12}$$

where S represents particle surface, $\sigma = -\varepsilon \boldsymbol{n} \cdot \nabla \Psi_1$ is the surface charge density, and $E_z = -\partial \Psi_2 / \partial z$ is the strength of the local electric field in the z direction. The hydrodynamic force acting on a particle in the z direction, F_D,

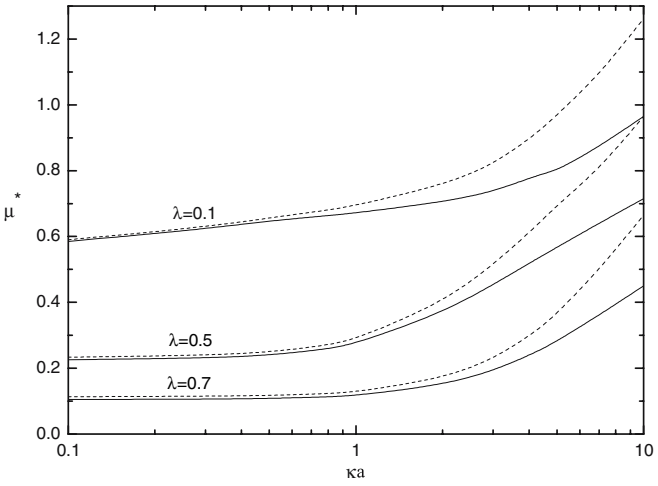

Fig. 3 Variation of scaled electrophoretic mobility μ^* as a function of κa at various λ. *Solid curves*, Newtonian fluid, *dashed curves*, Carreau fluid. n=0.8, $\alpha U/a$=0.8

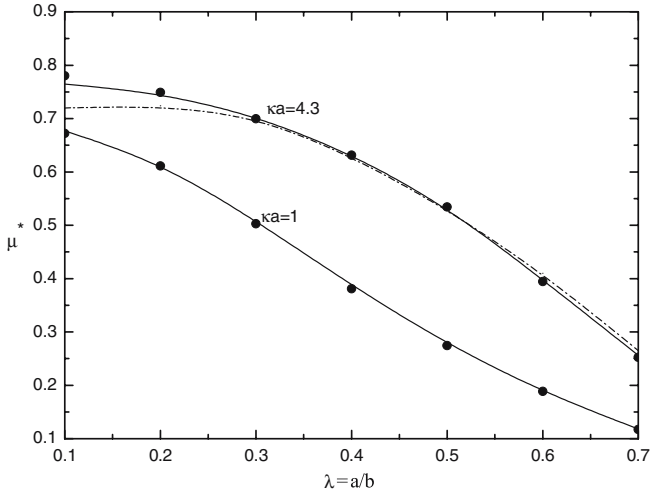

Fig. 2 Variation of scaled electrophoretic mobility μ^* as a function of $\lambda(=a/b)$ at two levels of κa for the case where a sphere of constant surface potential $\zeta_p = kT/e$ is placed on the axis of an uncharged cylindrical pore filled with a Newtonian fluid. *Solid curves*, from Hsu et al. [16], *dash–dotcurves*, from Shugai and Carnie [17], *discrete symbols*, present study

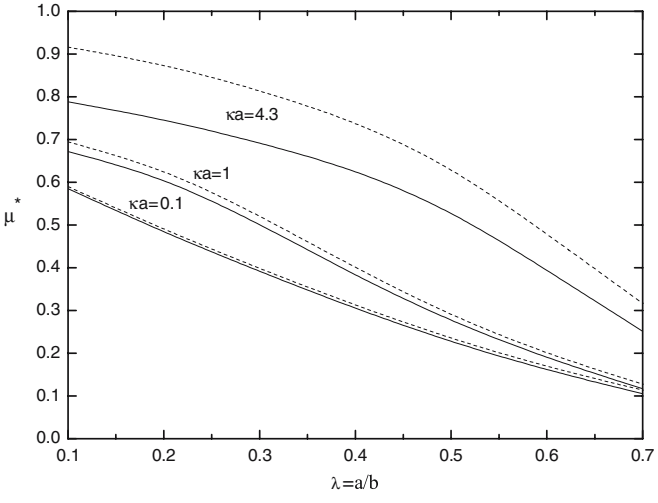

Fig. 4 Variation of scaled electrophoretic mobility μ^* as a function of λ at various κa. *Solid curves*, Newtonian fluid, *dashed curves*, Carreau fluid. n=0.8, $\alpha U/a$=0.8

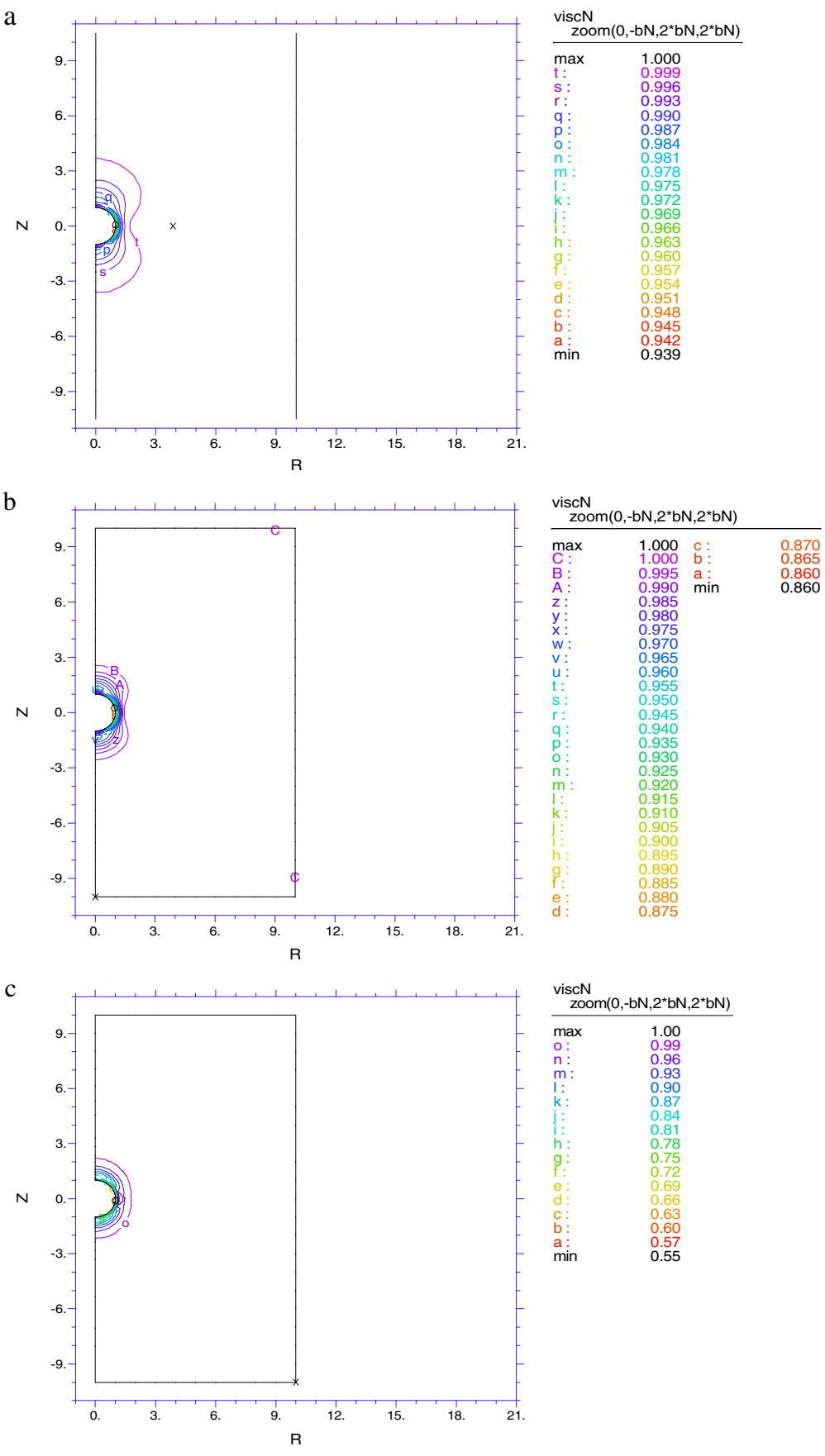

Fig. 5 Contours of viscosity at various κa for the case where λ=0.1. **a** κa=0.1, **b** κa=1, **c** κa=10. n=0.8, $\alpha U/a$=0.8

comprises the viscous force and the pressure. The former can be evaluated by [14]

$$F_D = \iint_S \eta \frac{\partial(\mathbf{u}\cdot\mathbf{t})}{\partial n} t_z dS + \iint_S -pn_z dS \tag{13}$$

where **t** is the unit tangential vector on particle surface, n is the magnitude of **n**, and t_z and n_z are the z components of **t** and **n**, respectively. The electrophoretic mobility of a particle can be calculated based on the fact that the net force acting on it in the z direction vanishes at steady state; that is,

$$F_E + F_D = 0 \tag{14}$$

FlexPDE [15], a commercial software that is based on a finite element method is adopted for the resolution of the governing equations and the associated boundary conditions. The applicability of this software is justified by applying it to the corresponding problem where the liquid phase is a Newtonian fluid. Figure 2 shows the variation of the scaled electrophoretic mobility $\mu^* = U\eta/\zeta_{ref}\varepsilon E$, $\zeta_{ref} = k_B T/e$ being a reference potential, as a function of the scaled radius of a particle $\lambda(=a/b)$ at different values of scaled double-layer thickness κa. The results of the works of Hsu et al. [16] and Shugai and Carnie [17] are shown. The former used the present numerical method, while the latter used Teubner's method [18]. According to Shugai and Carnie [17], their approach is inappropriate for small λ. On the other hand, the present approach does not have that limitation. Figure 2 indicates that the performance of the numerical method used in this study is satisfactory.

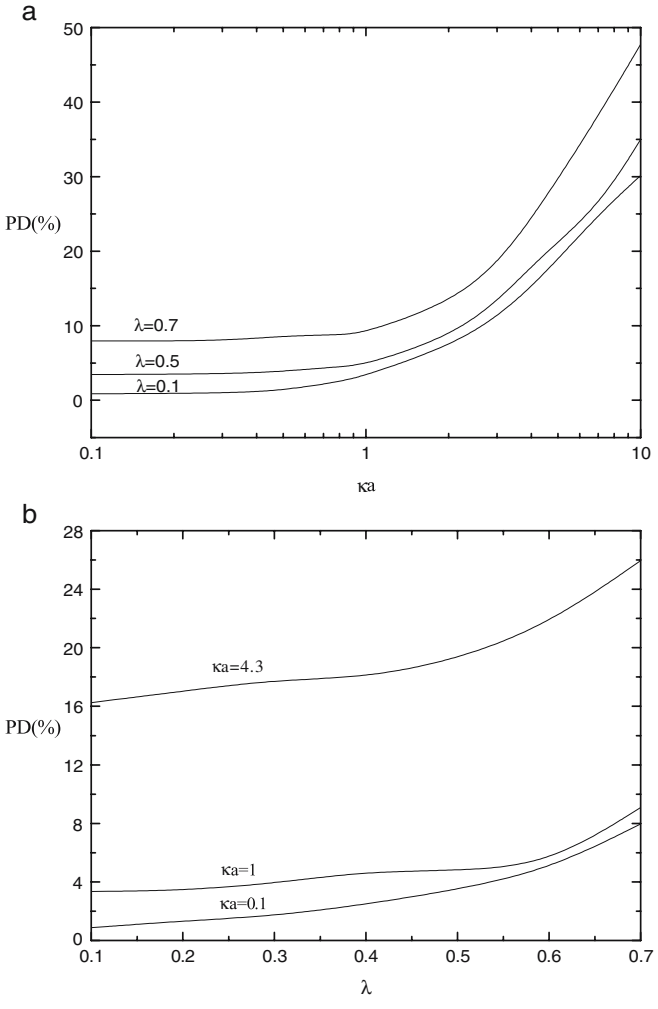

Fig. 6 Variation of percentage deviation in scaled mobility, $PD = 100\% \times \left(|\mu^*(Carreau)| - |\mu^*(Newtonian)| / |\mu^*(Newtonian)| \right)$ for the cases in Figs. 3 and 4

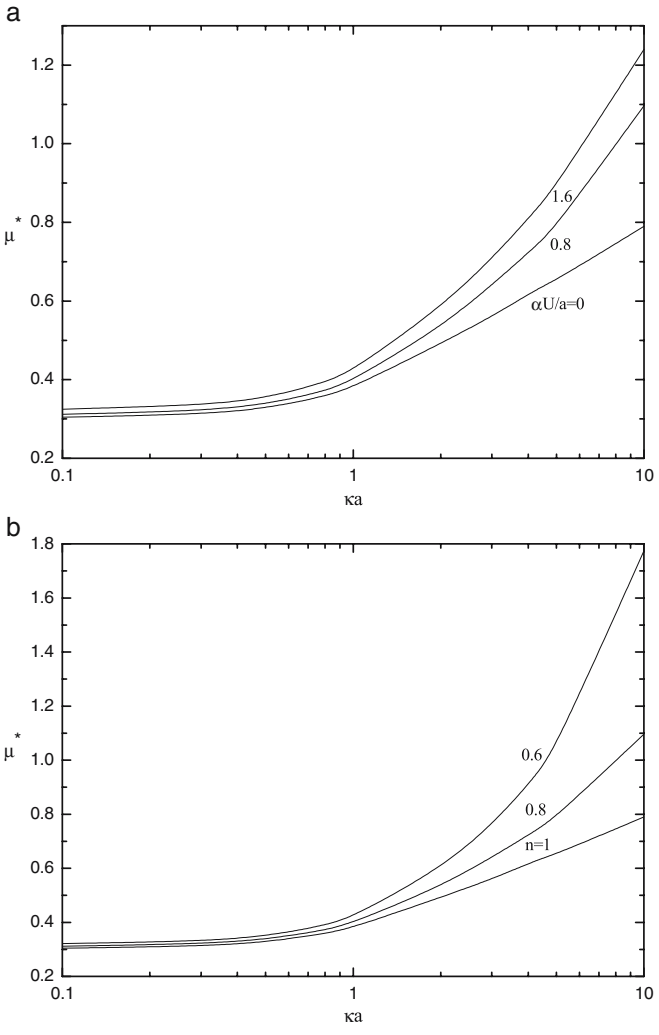

Fig. 7 Variation of scaled electrophoretic mobility μ^* as a function of κa for various values of $(\alpha U/a)$ at n=0.8 (**a**) and for various values of n at $\alpha U/a$=0.8, (**b**). Other parameters used are λ=0.4 and κa=1

Results and discussion

In the following discussions, the influences of the key parameters of the present problem, including κa, λ, and the parameters characterizing a Carreau fluid ($\alpha U/a$) and n, on the electrophoretic behavior of a particle are examined. For illustration, we assume that $\zeta_p^* = e\zeta_p/kT = 1$ and $\zeta_w^* = e\zeta_w/kT = 0$. The variation of the scaled electrophoretic mobility μ^* as a function of κa at various λ is presented in Fig. 3, while that as a function of λ at various κa is illustrated in Fig. 4. For comparison, the corresponding results for the case of a Newtonian fluid are also shown in these figures. Figure 3 reveals that for both Newtonian and Carreau fluids, μ^* increases monotonically with the increase in κa, but μ^* is more sensitive to the variation of κa in the latter. Also, for a fixed value of κa, μ^* (Newtonian) is smaller than μ^* (Carreau), and the difference between the two increases with the increase in κa, which is consistent with the observation of Lee et al. [4] for the case where a sphere is located at the center of a spherical cavity. The increase of μ^* as κa increases is typical to rigid particles and can be explained by the fact that the thinner the double layer surrounding a particle (larger κa), the larger the absolute value of the electrical potential gradient on its surface, the higher the surface charge density, and therefore, the greater the electric force acting on it. In our case, because particle surface is no-slip, a characteristic shear rate [4] $(U-0)/\kappa^{-1} = U\kappa$, which suggests that the larger the value of κa, the greater the shear rate and the more important the shear-thinning effect of the fluid, as is justified

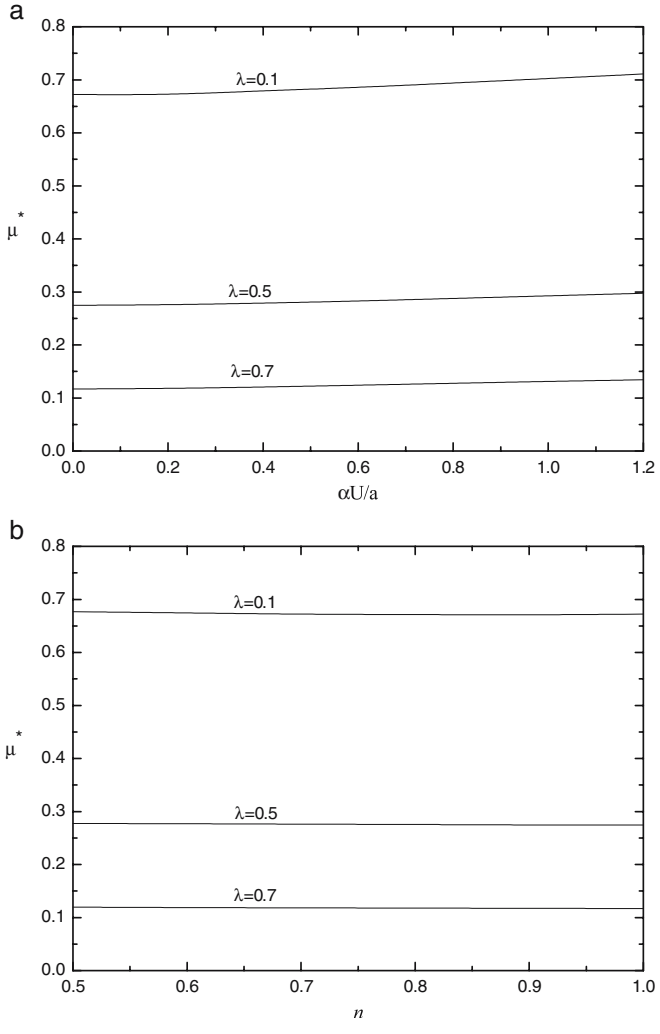

Fig. 8 Variation of scaled electrophoretic mobility μ^* as a function of ($\alpha U/a$) at various values of λ for n=0.8 and κa=1 (**a**) and as a function of n at various values of λ for $\alpha U/a$=0.2 and κa=1 (**b**)

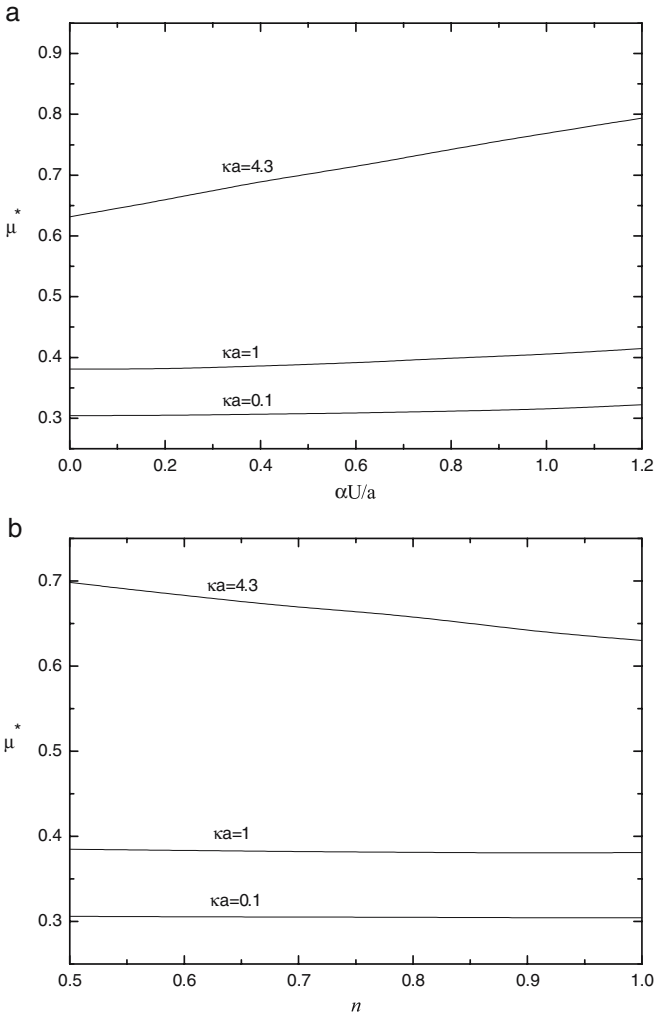

Fig. 9 Variation of scaled electrophoretic mobility μ^* as a function of ($\alpha U/a$) at various values of κa for n=0.8 and λ=0.4 (**a**) and as a function of n at various values of κa at $\alpha U/a$=0.2 and λ=0.4 (**b**)

in Fig. 5. This explains that the larger the value of κa, the greater the distance between μ^* (Newtonian) and μ^* (Carreau). For the range of λ examined, the influence of boundary on the electrophoresis of a particle is negative, as is illustrated in Fig. 4, where μ^* decreases with the increase in λ. Apparently, this arises from the effect of viscous retardation arising from the presence of the pore. Figure 5 shows that the minimum viscosity of the flow field occurs on the particle surface, and this minimum decreases with the increase in κa.

Let PD be the percentage deviation of the scaled mobility of the present Carreau fluid, μ^* (Carreau), from that of the corresponding Newtonian fluid, μ^* (Newtonian), defined by $PD = 100\% \times \left(\left| \mu^* \left(Carreau \right) \right| - \left| \mu^* \left(Newtonian \right) \right| \Big/ \left| \mu^* \left(Newtonian \right) \right| \right)$. The variation of PD for the cases in Figs. 3 and 4 is summarized in Fig. 6. This figure shows that PD increases with the increase in κa and/or λ, which is expected, and can be explained by the shear-thinning effect of the fluid. Note that, depending upon the values of the parameters assumed, the percentage deviation in the mobility of a particle can be in the order of 50%.

The inferences of the key parameters of a Carreau fluid on the scaled electrophoretic mobility of a particle μ^* are summarized in Figs. 7, 8, and 9. In general, the qualitative behavior of μ^* as κa varies is similar to that for the case of a Newtonian fluid [16]. Based on previous discussions, the general trends in Figs. 7, 8, and 9 are anticipated, including the fact that the more significant the shear-thinning effect of a fluid, reflected by a larger $(\alpha U/a)$ or a smaller n, the larger the μ^*, as well as the fact that the smaller the κa and/or the larger the λ, the smaller the μ^*. Note that the slopes of the curves in Fig. 9 are larger than those in Fig. 8, especially when κa is large, which implies that the influence of κa on μ^* is more important than that of λ on μ^*.

Conclusion

We analyzed the electrophoresis of a spherical particle along the axis of a cylindrical pore filled with a Carreau fluid under the conditions of weak applied electric field and low surface potential. This problem allows us to examine the influence of both the presence of a boundary and the nature of the liquid phase, two factors of practical significance, on electrophoresis. The results of numerical simulation reveal the following: (a) In general, the presence of the pore has the effect of retarding the movement of a particle, but the shear-thinning nature of the liquid phase is advantageous to its movement. (b) For both Newtonian and Carreau fluids, the mobility of a particle increases monotonically with the decrease in the thickness of double layer. In the latter, the mobility is more sensitive to the variation in the thickness of double layer. (c) The mobility of a particle in a Carreau fluid is larger than that in the corresponding Newtonian fluid, and the difference between the two increases with the decrease in double-layer thickness. Depending upon the values of the parameters chosen, the percentage difference in these mobilities can be in the order of 50%.

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